A Hierarchical Decomposition Approach for Railway Disruption Recovery

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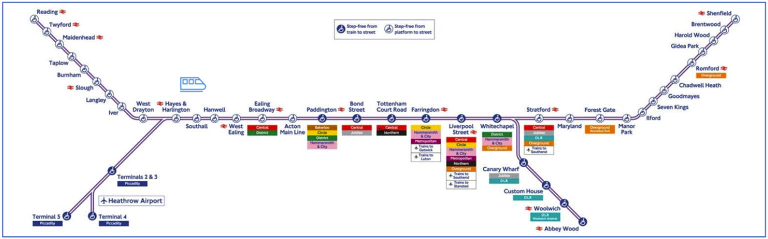
Motivation

- With rapid urbanization, railway systems in cities play a increasingly significant role in daily transportation.
- The railway system's normal schedules might be disrupted by unexpected events (e.g., train breakdown, bad weather).



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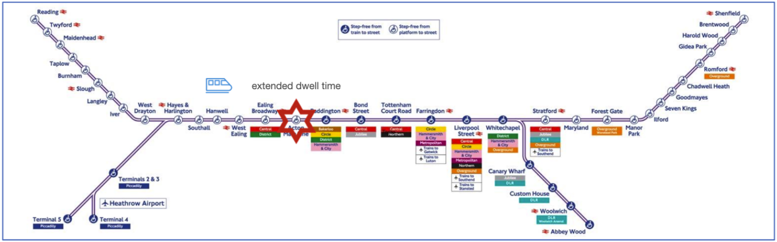
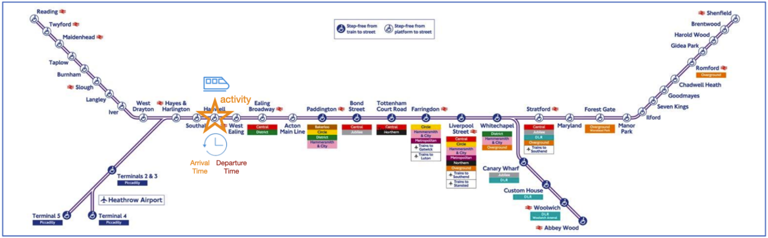


Problem Definition

A set of planned schedules assigned to trains, each of them consisting of:

- a direction (either “east bound” or “west bound”)
- a sequence of visited nodes, with an activity (either “stop” or “pass”), track usage, and arrival/departure times at every node



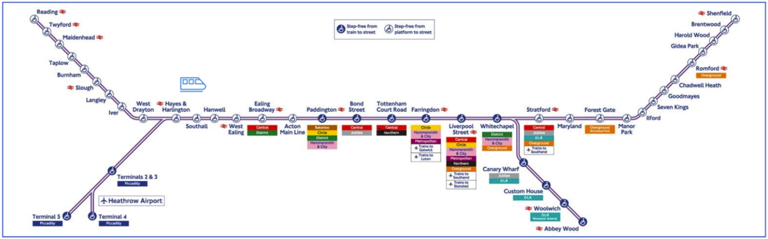
Operational Constraints

The railway operations have to respect the following three types of constraints:

I. rolling stock duty feasibility, including considerations such as minimum run time, minimum dwell time, changing end, etc.

II. minimum headway

III. minimum separation



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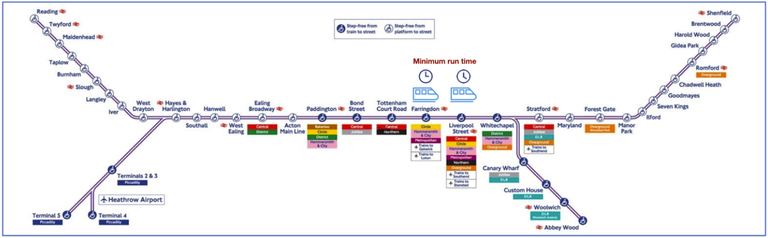

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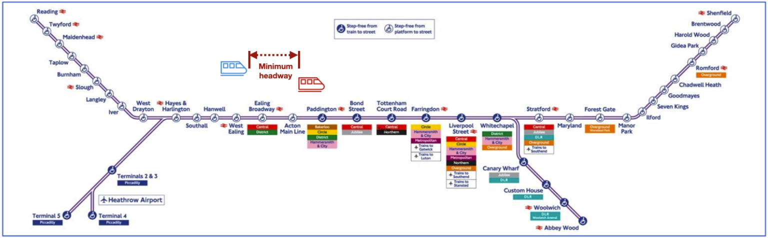

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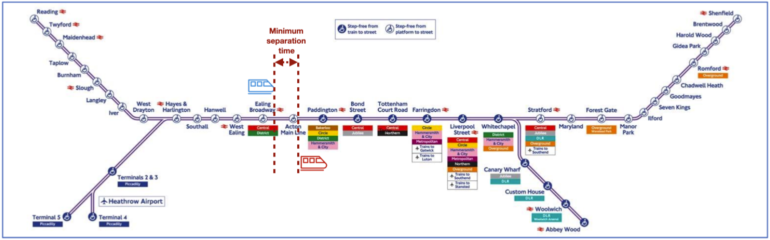

Problem Definition: Incidents

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A timetable may be disrupted by incidents such that:

- extended minimum run time
- extended minimum dwell time
- departure delay.

As a result, the planned schedules are disrupted.

How to efficiently identify a disruption recovery plan?

Decisions

Timetable amendments in order to restore feasibility:

- Rerouting trains: swaps, deadhead or repositioning of trains
- Course cancellation: cancel courses partially or fully
- Re-timing: adjust the planned arrival/departure times (prepone up to 5 minutes or postpone)
- Skipping stops: skips some planned stops

Objective

$$\text{minimize} \quad \underbrace{\text{Penalty}_{ss}}_{\textbf{stop-skipping}} + \underbrace{\text{Penalty}_{dd}}_{\textbf{destination delay}} + \underbrace{\text{Penalty}_{pf}}_{\textbf{passage frequency}}$$

Conceptual Modeling

$$\underset{\text{connection,timing}}{\text{minimize}} \quad \underbrace{\text{Penalty}_{ss}}_{\textbf{stop-skipping}} + \underbrace{\text{Penalty}_{dd}}_{\textbf{destination delay}} + \underbrace{\text{Penalty}_{pf}}_{\textbf{passage frequency}}$$

s.t. rolling stock duty feasibility

minimum headway

minimum separation

Hierarchical Decomposition

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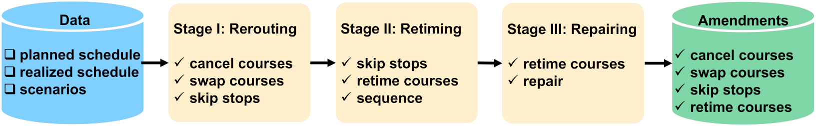
We propose a novel hierarchical decomposition approach:

I. **Rerouting:** Key decisions such as course cancellation and course-swapping have the most profound impact on the timetable amendments.

II. **Retiming:** This stage mainly concerns a proper adjustment of departure and arrival times for each course at each node.

III. **Repairing:** We design an efficient repairing procedure to attain feasibility of the returned solution.



Stage I: Rerouting

We consider to cancel courses partially or fully, and to swap courses if necessary.

Conceptual Modeling for Rerouting

$$\min_{\text{connection,timing}} \underbrace{\text{Penalty}_{ss}}_{\textbf{stop-skipping}} + \underbrace{\text{Penalty}_{dd}}_{\textbf{destination delay}} + \underbrace{\text{~~Penalty~~}_{pf}}_{\textbf{passage frequency}}$$

s.t. rolling stock duty feasibility

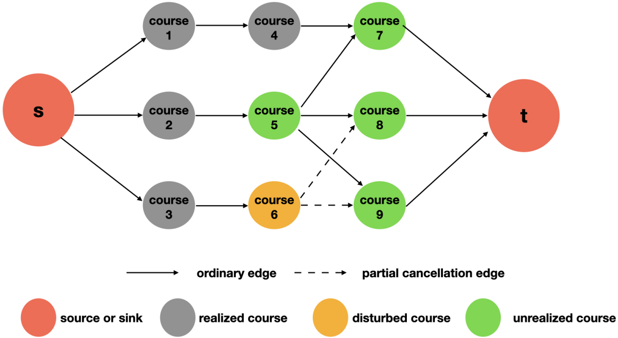
~~minimum headway~~

~~minimum separation~~

Two steps:

1. Resolve the feasibility issue for each individual course (stop-skipping is considered if it is economically preferable)
2. Connect courses to produce feasible rolling stock duties via a single-commodity flow model

Stage I: Rerouting



A single-commodity flow model

$$\min_{x,y} \quad \underbrace{\sum_{(i,j)\in\tilde{A}} c_{ij}x_{ij}}_{\textbf{partial cancel.}} + \underbrace{\sum_{c\in C} s_c y_c}_{\textbf{full cancel.}}$$

$$\text{s.t.} \quad \sum_{j\in\tilde{N}:(i,j)\in\tilde{A}} x_{ij} - \sum_{j\in\tilde{N}:(j,i)\in\tilde{A}} x_{ji} \begin{cases} \le |T|, & \text{if } i = s \\ = 0, & \text{otherwise} \end{cases} \quad \forall i \in \tilde{N} \setminus \{t\} \quad \text{Network flow}$$

$$\sum_{j\in\tilde{N}:(i,j)\in\tilde{A}} x_{ij} + y_c = 1 \quad \forall c \in C \quad \text{Course cancellation}$$

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in \tilde{A}$$

$$y_c \in \{0,1\} \quad \forall c \in C$$

Stage II: Retiming

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Once course connection decisions are made, one can now retime courses (i.e., the arrival and departure times).

Conceptual Modeling for Retiming

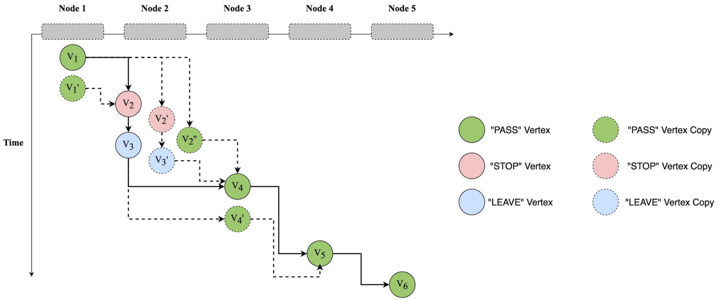
$$\min_{\text{~~connection~~},\,\text{timing}} \quad \underbrace{\text{Penalty}_{ss}}_{\textbf{stop-skipping}} + \underbrace{\text{Penalty}_{dd}}_{\textbf{destination delay}} + \underbrace{\text{Penalty}_{pf}}_{\textbf{passage frequency}}$$

s.t.
- rolling stock duty feasibility
- minimum headway (**approximate**)
- minimum separation

Stage II: Retiming

Using the solution obtained from the rerouting stage as a guide, a time-space network graph is built.

- Each activity (such as “stop”, “leave”, and “pass”) of a train at a specific station and a specific time point is denoted by a vertex.
- Fixed copies of a vertex are created to indicate this activity can be rescheduled earlier or later.



Stage II: Retiming

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A discrete-time MILP formulation is proposed.

$$\min_{x,y,z} \underbrace{\sum_{i \in V^{SS}} (c_1 y_i^1 + c_2 y_i^2 + c_3 y_i^3) \times b_i}_{\text{stop-skipping}} + \underbrace{\sum_{i \in V^{CE}} \sum_{j \in V} dd_i x_{ji}}_{\text{destination delay}} + \underbrace{\sum_{(i,j) \in RE} f_{ij} z_{ij}}_{\text{passage frequency}}$$

s.t.

$$\sum_j x_{ij} = \sum_k x_{ki} \quad \forall i \in V$$

Flow balance

$$\left.\begin{array}{ll} \sum_j x_{ij} = 1 & \forall i \in V^{TS} \\ \sum_{i \in V_d^{DS}} \sum_j x_{ij} = 1 & \forall d \in D \\ \sum_{i \in V_d^{DE}} \sum_j x_{ij} = 1 & \forall d \in D \\ \sum_{i \in V_c^{CS}} \sum_j x_{ij} = 1 & \forall c \in C \\ \sum_{i \in V_c^{CE}} \sum_j x_{ij} = 1 & \forall c \in C \end{array}\right\}$$

Degree

$$\sum_{i \in V_{h,f}^{SEP}} \sum_j x_{ij} + \sum_{i \in V_{h,b}^{SEP}} \sum_k x_{ki} \leq 1 \quad \forall h \in |MS|$$

Minimum separation

$$x_{ij} + \sum_l x_{kl} \leq 1 \quad \forall k \in V_{ij}^{TC}, (i,j) \in E$$

Track capacity

$$\sum_{(i,j) \in ec} x_{ij} \leq |ec| - 1 \quad \forall ec \in EC^{MH}$$

Minimum headway

Stage II: Retiming

$$
\left.\begin{array}{ll}
y_i^1 + y_i^2 + y_i^3 = \sum_j x_{ij} & \forall i \in V^{SS} \\
\sum_{i \in V_c^{SS}} y_i^1 \geq \sum_{i \in V_c^{SS}} y_i^2 & \forall c \in C \\
\sum_{i \in V_c^{SS}} y_i^2 \geq y_j^3 & \forall j \in V_c^{SS}, c \in C
\end{array}\right\}
$$

Skipping stops

$$
\left.\begin{array}{ll}
\sum_{i \in V_k^{RG}} \sum_j x_{ji} = \sum_l z_{lk} & \forall k \in RV \\
\sum_{i \in V_k^{RG}} \sum_j x_{ji} = \sum_l z_{kl} & \forall k \in RV \\
\sum_l z_{lk} \leq 1 & \forall k \in RV
\end{array}\right\}
$$

Degree for reference graph

$$
\begin{array}{ll}
x_{ij} \in \{0, 1\} & \forall (i, j) \in E \\
y_i^1, y_i^2, y_i^3 \in \{0, 1\} & \forall i \in V \\
z_{ij} \in \{0, 1\} & \forall (i, j) \in RE
\end{array}
$$

Stage III: Repairing

Both the course connection and station-visiting sequences are given, we now use a continuous-time model to

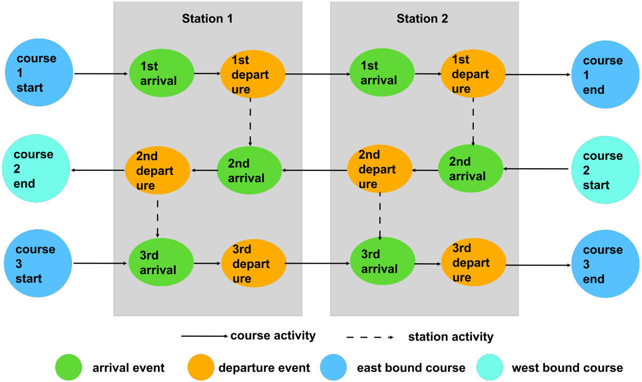
- fix the feasibility issue if minimum headway constraints are violated
- further reduce the penalty

Conceptual Modeling for Repairing

$$\min_{\text{~~connection~~, timing}} \quad \underbrace{\text{~~Penalty~~}_{ss}}_{\textbf{stop-skipping}} + \underbrace{\text{Penalty}_{dd}}_{\textbf{destination delay}} + \underbrace{\text{Penalty}_{pf}}_{\textbf{passage frequency}}$$

s.t.
rolling stock duty feasibility
minimum headway
minimum separation

Stage III: Repairing



Stage III: Repairing

$$\min \quad \underbrace{\eta p_d \sum_{c \in C^{OO}} dd_c}_{\textbf{destination delay}} + \underbrace{p_f \sum_{n \in N_f} \sum_{v \in V_n \cap V_{stop}} hg_v}_{\textbf{passage frequency}}$$

s.t.

$$a^{vc}_{v_i} - a^{v'_c}_{v'_i} \geq 0 \quad \forall n \in N \quad \forall v, v' \in V_n$$ **Sequence**

$$a^c_{i+1} - d^c_i \geq RT^c_{i,i+1} \quad \forall c \in C, i \in \{1, 2, \ldots, |N^c|\}$$ **Minimum run time**

$$a^{vc}_{v_i} - d^{v'_c}_{v'_i} \geq HT_{v,v'} \quad \forall e \in E, \forall v, v' \in V_e$$ **Minimum headway**

$$d^c_i - a^c_i \geq DT_{ci} \quad \forall c \in C, i \in \{1, 2, \ldots |N^c|\}, \text{ if } act^c_i = stop$$ **Minimum dwell time**

$$d^c_i - a^c_i = 0, \forall c \in C, i \in \{1, 2, \ldots |N^c|\}, \text{ if } act^c_i = pass$$ **Passing**

$$\left.\begin{array}{ll} d^{c'}_{|N^{c'}|} = a^c_1 & \forall t \in T, \forall c, c' \in C_t \\ a^c_1 = d^c_1 & \forall c \in C \end{array}\right\}$$ **Course time-connectivity**

$$\left.\begin{array}{ll} a^{vc}_{v_i} - a^{v'_c}_{v'_i} \geq \tau_{v_i} & \forall n \in N_d, k \in K_n, v, v' \in V_{n,k} \vee v' \in Ve \\ a^{vc}_{v_i} - d^{v'_c}_{v'_i} \geq \tau_{v_i} & \forall n \in N \setminus N_d, k \in K_n, v, v' \in V_{n,k} \end{array}\right\}$$ **Minimum separation**

$$a^c_{|N^c|} - pa^c_{|N^c|} - DD \leq dd_c \quad \forall c \in C^{OO}$$ **Destination delay**

$$a^t_{v_c, v_i} - a^t_{v'_c, v'_i} - hg_v \leq h_{f_t} \quad \forall n \in N_f, \forall f_t, \forall v, v' \in V_{nf_t} \cap V_s$$ **Passage frequency**

$$d^c_1 \geq pd^c_1 \quad \forall c \in C$$ **Course Start Time**

$$a^t_{ci} \geq curT \quad \forall c \in C, i \in \{1, 2, \ldots |N^c|\}$$

$$a^t_{ci}, d^t_{ci} \geq 0 \quad \forall c \in C, i \in \{1, 2, \ldots |N^c|\}$$

$$dd_c \geq 0 \quad \forall c \in C$$

- Java / Gurobi 9.5.2
- Intel 16-core i9-12900K CPU @ 3.2 GHz with 128 GB Memory

Instance	Greedy	This work	Percentage (%)	Ratio (%)	Time (sec.)
Incident_to_Heathrow	23,301,594	20,800	0/60/40	99.9	1821
Incident_ABWDXR	2,803,597	24,781	0/69/31	99.1	1165
Incident_inside_COS	26,184,411	378,953	82/12/6	98.6	2563
Incident_Ill_Passenger	25,923,814	45,315	0/69/31	99.8	1990

Our proposed hierarchical decomposition approach could **efficiently identify high-quality recovery solutions**.

Supporting Tools

1. Evaluator:
   - Input:
     - Instance data
     - Updated schedules
   - Output:
     - Penalties
     - Constraint violation
     - Analysis report for KPIs
2. Visualization system:
   - Gantt chart for each node, platform and link
   - Time-space network chart
   - Map

Supporting Tools